Insight into liquefaction by system identification

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This paper introduces system-parameter modelling, commonly known as system identification, to the estimation of dynamic soil properties. These methods are optimized for short and non-stationary signals. The Wildlife Site in California, subject to two large earthquakes (Elmore Ranch and Superstition Hills) on 24 November 1987, was used in an example analysis as the associated data are the only publicly available record of buried and surface motions. During the second, larger, earthquake the site soils liquefied. For the first time, both stationary and recursive system-parameter modelling methods were applied to the unique input–output data from the two earthquakes. The results show weakening of the soils system as pore pressure increases. Estimates of natural frequency, damping ratio and participation factor are given.

KEYWORDS: case history; earthquakes; in situ testing; liquefaction; statistical analysis; stiffness.

INTRODUCTION

An important goal for the geotechnical engineer is the ability to estimate soil properties without intruding into the soil mass. For the engineer interested in seismic behaviour of soils, the dynamic properties of the soil are of interest, particularly changes due to large strains. The archetype large-strain field excitation is earthquake strong motion. Ideally, both soil motions into the soil layer of interest and soil motion on the surface above the layer are recorded. Given this known input propagating upwards from depth, and the output at the top of the soil column, the behaviour of the soil can be modelled by inverse theory. If a suitable model is chosen to represent the system of interest, the estimated model parameters will correspond to important mechanical parameters of the system, such as damping, natural frequency and stiffness. This estimation of parameters is commonly known as system identification (SI).

The traditional method of geotechnical analysis of dynamic soil motions is through the Fourier transform. However, serious problems arise when this method is applied to short data streams, and to signals changing through time—non-stationary signals. This study was undertaken to show the effectiveness of a different type of model, a system-parameter model commonly used in automatic control and geophysics, and called system identification avoids many of the limitations inherent in calculating the system transfer function by Fourier techniques. An important aspect of particular system-parameter models is the theoretical link between the estimated system parameters and the mechanical parameters of a lumped-mass oscillator. To avoid confusion with ‘parametric studies’, this method of estimating system parameters is referred to below as ‘system-parameter modelling’. The system-parameter model allows estimates of system dynamic properties to be made if an input–output data set is available.

This paper applies both stationary and time-varying system-parameter models to the input–output data sets recorded during the two 1987 earthquakes (Elmore Ranch and Superstition Hills) at the Wildlife Site in the Imperial Valley, California. Since the inverse problem (identifying a system by its effect on a signal) is non-unique,
it is necessary to understand the mechanistic behaviour of the system being modelled in order to facilitate selection of the correct model. To this end geotechnical, geological, and seismological analyses of the Wildlife site and earthquake events are presented.

This paper starts by introducing and explaining system-parameter modelling in the context of SI. The Wildlife site is then introduced and physical properties examined. The geotechnical behaviour of the site during strong shaking is examined. Finally, the SI techniques are applied to the Wildlife site data and system parameters are estimated.

SYSTEM IDENTIFICATION

System-parameter modelling

The process of inversion allows the estimation of the system transfer function (filter) if the input and output signals are known. A simple model for characterizing a system is as a parametric relationship between system input and output. Such a model, referred to as an autoregressive moving average (ARMA) model, is based on discrete time series analysis

\[ y_i = a_1 y_{i-1} + a_2 y_{i-2} + \ldots + b_0 x_i + b_1 x_{i-1} + \ldots \]

\[ \equiv \left( \sum_{j=0}^{na} b_j x_{i-j} + \sum_{k=1}^{nb} a_k y_{i-k} \right) \] (1)

where \( y_i \) is the actual output data sequence, \( x_i \) is the input sequence (assume white noise for simple spectral estimation), \( na \) and \( nb \) are the AR and MA orders respectively, and the subscript is the time step counter. The output is seen as a combination of the input history acted on by the \( b \) coefficients plus the past outputs acted on by the \( a \) coefficients. The input series, involving the \( b \) coefficients, is a causal MA process (convolutional). The series involving weighted past output values (\( a \) coefficients) is a non-causal AR process. The lengths of the AR and MA processes (model order) must be explicitly chosen so that the model best represents the process.

Applying the shifting theorem to equation (1) yields the Fourier transform (Bracewell, 1978)

\[ Y(w) = X(w) \left( b_0 + b_1 e^{i\omega} + b_2 e^{2i\omega} + \ldots \right) \]

\[ = X(w) \left( \frac{b_0}{1 - a_1 z^{-1} - a_2 z^{-2} + \ldots} \right) \] (2)

where \( i = \sqrt{-1} \) and \( \omega \) is circular frequency. Applying the \( Z \)-transform (Bracewell, 1978), where \( z = e^{i\omega} \), and rearranging, yields the frequency domain transfer function \( H(w) \)

\[ H(w) = \frac{Y(w)}{X(w)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots}{1 - a_1 z^{-1} - a_2 z^{-2} + \ldots} \] (3)

The ARMA model is very powerful in that it can easily model sharp drops, sharp peaks, and smooth spectral behaviour. It is also the most parsimonious estimator (Robinson, 1982), describing a complex process with very few parameters calculated from a small length of data. System-parameter modelling avoids many of the difficulties inherent in the traditional Fourier methods, discussions of which can be found in many books and journals (e.g. Glaser, 1993, 1995; Johansson, 1993; Pandit, 1991). Extensions of this model, e.g., ARMAX, ARX and Box-Jenkins, allow input, system, and output noise to be expressly modelled (Ljung, 1987). In particular, the ARX model includes the effect of uncertainties and noise as a white noise term.

The ARMA model has special significance since it can be derived directly from the differential equation of motion for an \( N \)-degree-of-freedom (DOF) system, with the damping ratio and resonant frequency as the model parameters (e.g. Safak, 1988; Gersch & Luo, 1972). Equation (1) was originally derived by Yule (1927) for a damped oscillator. The equation makes a simple linearization of the system parameters through the AR and MA polynomial weights. Equation (1) is the associated ARMA difference equation, which also is the difference equation of the integral of the equation of motion of an \( N \)-DOF system (Ghanem, Gavin & Shinozuka, 1991).

A \( 2n-2n \) ARMA model is therefore a valid model for a layered soil system, or soil-structure interaction problem. The damping ratio and resonant frequency of the \( N \)-DOF oscillators are contained in the \( 2n \) AR parameters. Phase relations are preserved in the MA parameters. The modal frequencies \( \omega \), percentage of critical damping \( \xi \) (Ghanem et al., 1991) and power participation factor \( p_j \) (Pandit, 1991; Safak, 1988) are calculated from the system poles and residues found from partial-fraction expansion of equation (3). The modal parameters are defined as

\[ \omega_j = \frac{\sqrt{\lambda_j^2 + \delta_j^2}}{\Delta t} \] (4)

\[ \xi_j = \frac{\delta_j}{\sqrt{\lambda_j^2 + \delta_j^2}} \] (5)

\[ p_j = -r_j \text{ conj}(z_j) - z_j \text{ conj}(r_j) \] (6)

where \( \lambda = \text{Arg}(z_j) \), \( \delta_j = -(0.5) \text{ln}|z_j|^2 \), \( z_j \) is the pole for mode \( j \), \( r_j \) is the residue for mode \( j \), and \( \Delta t \) is the inverse of the digitization rate.

Adaptive (recursive) model estimation

Traditional methods of system estimation, both parametric and non-parametric, are strictly valid only for stationary data. A stationary signal is one
whose statistics do not change with time. The commonly invoked, loose definition of stationarity requires that the variance of the signal be constant over any and all time windows. Inherent in the Fourier transformation of a time series to the frequency domain is the averaging of the signal components over the sampling period \( T \). A piece of time is frozen over this period and the assumption is made that all time before and after is the same, i.e. repeated forever. The energies present at each component frequency are integrated over the entire time period \( T \).

The difficulty with non-stationary signals is that these energies are changing during this period. If the frequencies present are changing over this time window, the resulting estimation, regardless of the method used, will be a smeared average as if all the frequencies with a given energy were active throughout the entire period. For weakly non-stationary processes, the effect over a small time period is unimportant. If needed, the signal can be cut into relatively stationary sections and spectra found using methods specially designed for short data segments, i.e. Burg’s method.

The field of adaptive filtering was formed to model non-stationary processes. As the statistics of the signal change through time, the filter ‘adapts’ to the changing variance with new parameters that reflect the structure of the system at that point. The predicted value for the next time step can be compared with the actual value, and the difference (referred to as innovations)

\[
(y_t - \hat{y}_t)^2 \tag{7}
\]

where \( y_t \) is actual output at time \( t \) and \( \hat{y}_t \) is the prediction of output at time \( t \) made at time \( t-1 \), will give a measure of how well the filter is doing its job. The term ‘innovations’ is used because this information is new information that cannot be predicted by the model at this particular step.

AR parameters can be sequentially estimated so that the parameters are adaptive to the changing nature of the process (Marple & Lawrence, 1987). The AR parameters are updated after each data point, tracking slowly non-stationary signals. A forgetting factor, commonly a damped negative exponential, is used so that older data carry less and less weight. A frequency domain estimation can be made at any time step by evaluating the AR parameters around the unit circle, giving the spectral representation of the behaviour of the process at that time.

The most popular direct adaptive filter, or process model, is the so-called Kalman filter (Kalman, 1960; Kalman & Bucy, 1961). Sorenson (1970) points out that the Kalman approach is a direct descendant of Gauss’s least squares, except now neither the signal nor the noise model must be stationary—the state may change from sample point to sample point. Nau & Oliver (1979) state that the Kalman filter is based on a dynamic AR model defined by ‘two concurrent random equations of motion’

\[
x_t = H_{t-1}^T \Phi_t + a_t \tag{8}
\]

the AR(p) equations of motion, and the ‘motions’ of the parameters

\[
\Phi_t = \Phi_{t-1} + b_t \tag{9}
\]

where \( p \) is the number of prior observations utilized, \( H_{t-1} \) is a vector of \( p \) prior data observations \( x_t, x_{t-1}, \ldots, x_{t-p} \), \( \Phi_t \) is a vector of \( p \) AR parameters, \( a_t \) is Gaussian white noise with mean 0 and variance \( \sigma^2 \), and \( b_t \) is Gaussian white noise with 0 mean and covariance matrix \( Q \).

Equation (9) estimates a value of \( \Phi_t \), comprising \( p \) previous parameters, through a random walk equation. The estimate uses the weighted \( p \) previous data points, and yields a new observation \( x_t \) when added to a new noise value. The least squares solution solves the equations so that the innovations (equation (7))—new, dynamic information that cannot be predicted—are minimized in a least squares sense each time step.

The theory behind the Kalman filter can be manipulated to yield the system parameters for the case where there is no a priori information about the noise, and even when there is no information about the input signal. The so-called extended Kalman filter has been applied very successfully to non-stationary (and non-linear) estimation problems (Ljung, 1979; Astrom and Eykhoff, 1971). The manner of application is actually straightforward. The Kalman model is constantly updating its estimation of the dynamic process by examining the innovations. The dynamics can be due to a changing input or noise process, or it can be due to the system itself changing. The effect is a linearization between single time steps, but if the system is changing slowly compared with the time step used, the linearization is ‘invisible’ and the non-linear behaviour is well modelled.

Analysis methods applied to the soil system

For the problem at hand, estimating the soil parameters during the Superstition Hills earthquake at the Wildlife site, both the system input and output are known. The data were initially analysed to determine whether they could be modelled as stationary segments. A recursive segmentation scheme, which attempted to break the data into segments with a chosen maximum variance (Ljung, 1987), did not work well since it was not possible to determine the ‘correct’ variance a priori. Instead, a more direct method was used—the output
simulated by the calculated system had to accurately model the actual measured output. The input-output data record was broken into segments based on a mechanistic understanding of the seismic event (discussed in the following sections). Initially it was assumed that the various segments were basically stationary. If the stationary model could not accurately and parsimoniously simulate the segment output, a non-stationary recursive model was used. In addition, the appropriateness of the model was checked by ensuring 99% confidence in both the whiteness of the residual autocorrelation function and the cross-correlation function between the input and output residuals (Bohlin, 1987). A final verification of model suitability was made by examining the ability of the parameters to model independent input-output data from the E-W Superstition Hills and the complete Elmore Ranch time histories.

The stationary algorithm uses a least squares estimation for the ARX model. It is necessary to estimate the number of parameters to be calculated, which is essentially estimating the DOF of the soil system. There is no obvious answer to the DOF of the system, so several verification techniques were employed to ensure that a proper model order was estimated. Model order was increased in $2n-2n$ (ARMA parameters) steps, each step corresponding to adding an additional degree of freedom $n$ to the model. The simulated output of the model was then compared with the actual output for congruence, and the smallest number of parameters needed to accurately characterize the system was chosen as the system model order. Examination of the pole and zero plot ensured that excessive, overlapping parameters were not included (Ljung, 1987). If the segments proved non-stationary, they were analysed using a recursive Kalman filter technique that expressly accounts for non-stationarity (Ljung, 1987).

Software used for SI analysis
The analyses undertaken for this paper was made in the MATLAB interactive environment. MATLAB is a matrix-based system which evolved from the LINPACK and EISPACK libraries commonly used for mainframe FORTRAN numerical analysis. Complex numerical problems can be speedily solved without programming in the traditional sense. Add-on toolboxes are available to provide specific functionality for applications in digital signal processing, automatic control systems, non-linear simulation, neural nets, optimization and statistics.

For this paper, the standard routines contained in the MATLAB System Identification Toolbox (MathWorks, 1991) allowed SI to be used as a tool accessible to the geotechnical engineer. Virtually every approach and algorithm encountered in the literature could be duplicated rapidly and accurately. When run on an SGI Indy workstation, all aspects of the analysis were quick enough to be interactive.

THE WILDLIFE SITE IN THE IMPERIAL VALLEY
Introduction and geography
Two large earthquakes, Elmore Ranch and Superstition Hills, occurred on 24 November 1987. The shaking was strong enough to induce liquefaction at the Wildlife site, which suffered more than 2 m of lateral spread towards the Alamo river, and sand boils occurred over at least 33 ha (Holzer, Youd & Hanks, 1989). Accelerograms from buried and surface transducers, as well as pore pressure-time histories, were captured for both the Elmore Ranch and Superstition Hills earthquakes (Brady, Mork, Seekins & Switzer, 1989). This is the only full input-output data set known to exist for a site experiencing liquefaction, and the only set of records complete enough to allow an unambiguous SI analysis.

The Wildlife site is located in the western Imperial Valley, 13 km north of Brawley, Imperial County, California, as shown in Fig. 1. It is located in the Imperial Wildlife Management Area, on an incised flood plain of the Alamo river (Bennett, McLaughlin, Sarmiento & Youd, 1984) approximately 32 km east of the epicentre of the Superstition Hills earthquake. The deposits consist of various flood plain, fluvial and lacustrine materials with seven distinct units in the first 26 m. The plan view of the array is shown at the top of Fig. 2, while the bottom of Fig. 2 presents the physical relation of the piezometers to the soil units. Two piezometers were installed in subunit B1, three in B2, and one in the dense clayey silt of unit D.

Geotechnical description
The pertinent geotechnical properties of the first 13 m of soil at the Wildlife are shown in Fig. 3. The water table, controlled by the nearby Alamo river, was approximately 1.2 m below the surface when the Superstition Hills earthquakes struck. Unit A consists of interbedded sandy silt and clayey silts deposited by periodic flooding of the nearby Colorado river. The upper 1 m of unit B, B1, is a very loose, poorly graded, silty sand which phases towards the medium dense silty sand of subunit B2. In general, the materials in unit B grade from coarsest at the bottom (7 m) to finest at the top. Brady et al. (1989) note that a wood
Fig. 1. Location of the Wildlife Site; locations of the Elmore Ranch (M, 6.2) and Superstition Hills (M, 6.6) earthquakes indicated by *.

A fragment recovered from a depth of 6 m was estimated to be about 230 years old by carbon-14 analysis. Unit B is believed to be a point-bar deposit, on the concave side of a meander curve (Bennett et al., 1984). Unit C ranges from a medium-to-stiff clayey silt to a very stiff silty clay. Unit D is a dense, well-graded silt, cemented towards the top of the unit.
Pore pressure history

Analysis of pore pressure histories recorded during the Elmore Ranch and Superstition Hills earthquakes yields additional understanding of the recorded strong motion signals, and thus simplifies and strengthens later interpretation. Complete pore pressure records were made during both temblors at five different depths (Brady et al., 1989). Records show that an increase in pore water pressure was measured during both the Elmore Ranch and Superstition Hills earthquakes. The Superstition Hills records have been of greater interest, since for this excitation the pore pressures increased enough to cause liquefaction. This was proved by the appearance of sand boils at the Wildlife site after the Superstition Hills temblor (Holzer et al., 1989).

This paper focuses on the pore water pressure history of piezometer P5, which was buried at a depth of 2.9 m, at the top of soil layer B1. This transducer is being singled out for several reasons: analysis of sand boil ejecta after the 1981 Westmorland earthquake indicated that unit B1 liquefied (Holzer et al., 1989); P5 most clearly indicated that the excess pore pressure exceeded the total overburden pressure; it was located at the top of the layer expected by the experts at the USGS to liquefy during the Superstition Hills shaking and therefore (by some theories) would be at the point of initial liquefaction (Scott, 1986; Florin & Ivanov, 1961); and it is believed that P5 is the transducer in the array most likely to have performed correctly (Hashmand, Scott & Crouse, 1991, 1992).

Fig. 2. Layout of instrumentation at the Wildlife Site (Bennett et al., 1984)
Figure 4 compares the first 40 s of buried north-south acceleration (Brady et al., 1989) with the pore pressure ratio $r_u$ (ratio of excess pore pressure $u$ to initial effective overburden stress $\sigma_k$) history for piezometer P5, Superstition Hills earthquake. At time $t_0$ (subscript = time from first arrival in seconds) the initial shear wave (S-wave) energy from the first subevent arrived, and a very slight increase in pore pressure was recorded by P5. At approximately time $t_{31}$ energy from subevent 2 excited the accelerometer, and the pore pressure started to increase at a steady rate. Finally, at time $t_{68}$ strong shaking was triggered by the arrival of subevent 3, and the pore pressure ratio at P5 increased at a rapid rate to a maximum greater than 1.2. After $t_{68}$ the rate of increase of $r_u$ was relatively constant for the duration of source motion (~ 7.5 s) until it reached 0.6.

**Geology**

The Imperial Valley is a structural depression caused by the active spreading of the Gulf of California (Magistrale, Jones & Kanamori, 1989). The surface sediments in the areas of interest are Holocene lacustrine silty sand and claystone,
interbedded with alluvial deposits, below which are slightly consolidated Pleistocene silty and clayey lake deposits with sand and gravelly units (Sharp et al., 1989; Bennett et al., 1984). The impedance contrasts (i.e. stiffness ratio) of the geology are such that the vertical motions due to a local earthquake are primary waves (P-waves), while the horizontal motions are manifestations of the passage of S-waves (Wald, Helmberger & Hartnell, 1990). This assertion is borne out by the phase shift that occurred between the buried and surface horizontal motion records and by the lack of shift in the vertical records.

**Seismology**

On the evening of 23 November 1987, an earthquake occurred along the previously unknown Elmore Ranch fault, with a shear magnitude $M_s$ of 6.2 and a hypocentral depth of about 11 km (Magistrale et al., 1989). The maximum measured surface horizontal displacement was 125 mm, which occurred during the event itself. There has been no measured movement along the zone for over 2 years after the initial displacement in 1987 (Sharp et al., 1989).

Twelve hours after the Elmore Ranch earthquake, a large (6.6$M_s$) long-duration earthquake struck on the Superstition Hills fault (Magistrale et al., 1989). The Superstition earthquake was a complicated event characterized by an extended period of strong motion (Wald et al., 1990). Fig. 5 shows the acceleration time history in the north–south direction, recorded at a depth of 8 m. There appear to be arrivals of energy at approximately 3 and 9 s following arrival of the initial S-wave. Thorough and independent study of the strong motion and teleseismic records by several researchers indicates that the overall earthquake event was made up of three distinct subevents which must have occurred over an extended area (Hwang, Magistrale & Kanamori, 1990; Wald et al., 1990; Frankel & Wennerberg, 1989). While all three studies reached essentially the same general conclusions, the study by Wald et al. was the most exhaustive and their quantitative results are used in this paper. The physical location of these subevents is shown in Fig. 6 in both the map and the plan view.

Wald et al. (1990) calculated the arrival times relative to initial energy arrival, seismic magnitude and horizontal extent of the three subevents, which are presented in Table 1. While the $M_s$ values of the three subevents are very similar, the energy released by subevent 3, characterized by the seismic moment, is almost three times larger than the combined energy release of the first two subevents. The source durations of the first two subevents were short (less than 2 s for subevent 2), while it was estimated that subevent 3 required about 7.5 s for the displacement to propagate the 18 km (Wald et al., 1990). The calculated arrival

![Graph](image_url)

*Fig. 5. Acceleration record at a depth of 8 m, N–S direction, Superstition Hills earthquake*
times of the three subevents are shown in Fig. 5, exactly matching the visually perceived arrival of packets of energy.

SYSTEM IDENTIFICATION OF THE WILDLIFE SITE

For the SI analyses, the input motion comes from the buried accelerometer (SM1) and the output motion is from the surface accelerometer (SM2). The location of these sensors in relation to the soil profile is shown in Fig. 2. While the estimated models are transfer functions representing the entire soil system between the two accelerometers, a simple model for the soil system permits the focusing estimation of soil properties to the liquefying soil layer between the two accelerometers. The soil profile of the Wildlife site above layer B1 is relatively uniform, and the particle motion due to S-waves is geologically restrained to the horizontal. In addition, the pore-pressure history (Brady et al., 1989) indicated that location P5, at the top of layer B1, was the point of initial liquefaction. Once the pore pressure in layer B1 begins to rise, the sand in this layer is very soft in comparison to the rest of the system. Therefore, the motion of the surface accelerometer relative to the buried accelerometer essentially reflects the behaviour of the liquefying soil from layer B1 with the soil mass above moving as a whole on the liquefying sand. Prior to softening, the modal behaviour of the soil system will be dominated by the weakest soil between the two geophones, which is within the top 3.5 m.

The goodness-of-fit between the estimated time history and the actual time history is given by a normalized root mean square (RMS) estimate of errors. Since the RMS estimate is a function of the peak amplitudes of the signal, a normalized RMS was taken for each event, with each event normalized to a peak velocity of 1 cm/s. This allows simple comparison of errors between segments of different shaking amplitudes. It should
be noted that for all segments analysed, the peak positive and negative velocities estimated were a
total one-to-one fit to the actual.

**Pseudo-stationary approach**

The simplest method of analysis is to try to segment the site time history into pseudo-stationary pieces. The input-output data record was initially broken into segments based on the mechanistic understanding showing the events to comprise three subevents. Segment 1 (see Fig. 5) is the segment from the start of the time history to the arrival of subevent 2 approximately \(t_{5.8}\) s later (171 data points). Segment 2 runs from the arrival of subevent 2 to the arrival of subevent 3, approximately \(t_{5.8}\) s thereafter (150 data points). The segment corresponding to the shaking due to subevent 3 is very non-stationary due to the energy of the input signal and the rapid rise of pore pressure, and could not be analysed as stationary. Segment 4 encompassed the coda of the signal, after the shaking stopped at approximately \(t_{3.0}\) s into the quake, to the end of the record at 93 s (1870 data points).

Each of the first two segments, corresponding to the beginning of the record up to the arrival of subevent 3, can be accurately modelled by a stationary ARX model. Segment 1 is best represented as a 4-DOF system, with a normalized RMS error between simulated and actual velocity of 0.04. Segment 2 proved to be representable as a 3-DOF system, with a normalized RMS error between simulated and actual velocity of 0.08. A comparison of the spectral estimates of the first two segments is shown in Fig. 7, along with the estimates of modal parameters. Table 2 presents a summary of the ARX-calculated modal parameters, where it is seen

![Graph showing spectral amplitude vs frequency with various frequencies and damping ratios marked.](image)

**Fig. 7. Stationary spectral estimates for the first and second segments, with estimates of natural frequency, damping ratio and participation factor.**

**Table 2. Summary of system parameters estimated for pertinent times during the Superstition Hills earthquake**

<table>
<thead>
<tr>
<th>Time: s</th>
<th>1st mode</th>
<th>2nd mode</th>
<th>3rd mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f ) Hz</td>
<td>( \zeta )</td>
<td>( p )</td>
</tr>
<tr>
<td>(-4-2.8^*)</td>
<td>3.7</td>
<td>4</td>
<td>0.82</td>
</tr>
<tr>
<td>(2.8-8.8^*)</td>
<td>2.4</td>
<td>87</td>
<td>0.26</td>
</tr>
</tbody>
</table>

* ARX (stationary) estimate using combined model.
that the natural frequency diminished slightly after the arrival of subevent 2, an indication of reduction of system shear modulus.

The change in the soil system was so slight during the initial 8.8 s of excitation that it proved possible to model the combined segment accurately with one 4-DOF ARX model (normalized RMS error = 0.04), with the characteristics of the second subevent overwhelming the lower energy first arrival. The availability of the input–output seismic history of the site due to the earlier Elmore Ranch temblor allows an independent verification of the validity of the estimated model. The suitability of this model is proven by comparing the actual output surface velocities from the Elmore Ranch N–S horizontal record with the simulated surface output from the 4-DOF ARX model. The congruity between simulation and real life is shown in Fig. 8. The signals are virtually identical, with a normalized estimation RMS error of only 0.09. The identified model performed equally well on all other directional records available for this site.

Evolutionary estimates of soil behaviour

In order to illustrate the time-varying behaviour of the soil (due to the time-varying pore pressure), the initial 40 s of the Superstition Hills record was analysed with a recursive technique employing a Kalman filter model with 3-DOF. In this analysis the behaviour of the soil from its initial state to the end of all strong shaking and complete liquefaction was taken into account. The three modal natural frequencies, calculated from the six ARMA parameters through time, are shown in Fig. 9. The first mode frequency drops considerably with the arrival of energy from the very large third subevent. A comparison of modal participation, presented in Fig. 10, shows that most of the energy was dissipated through the first mode.

The evolution of system damping through time, and for each mode, is presented in Fig. 11. It is seen here too that most of the energy is dissipated through the first mode. The rapidly changing system is reflected in the seemingly wild fluctuation of mode 1 damping after the arrival of subevent 3. Note that the instability in mode 3 damping dies down at this point.

Fig. 9. Evolution through time of the natural frequency of the modelled soil, initial 40 s of Superstition Hills earthquake
Fig. 10. Evolution through time of the modal participation, initial 40 s of Superstition Hills earthquake

Fig. 11. Evolution through time of the modal damping, initial 40 s of Superstition Hills earthquake

Some verification of the validity of this model and its ability to capture the time-varying behaviour of the soil system can be had by examining how well it can simulate the actual surface velocity response. The comparison between the actual and simulated surface response is shown in Fig. 12. The calculated model was able to simulate surface velocity history with an RMS error of 2.01. The model appears to capture the behaviour of the system throughout the liquefaction process very well indeed.

Initial visual inspection of the final coda segment, corresponding to the inertial soil motion after the end of source energy, indicated that the input-output signal had a regular oscillatory pattern and would therefore be easy to model as a low-order ARX system. However, this segment proved to be quite non-stationary due to the constant increase in phase shift between buried and surface signals. Application of a 3-DOF Kalman filter captures the essence of the soil response (normalized RMS error = 0.70). For this segment, the system parameters are transformed into the frequency domain for easier comparison with traditional Fourier-based response functions. The evolutionary spectra for the Coda segment are presented in Fig. 13, with smoothed power spectra for each time step combined into a time-dependent surface. As indicated by the time-dependent values for $\zeta$, $f$, $p$ summarized in Table 3, the soil system is slowly changing as the pore-water pressure comes to equilibrium. The increase in damping as the sand begins to settle out of the viscous sand/water fluid and reconsolidate is responsible for dissipating a large part of the energy at this point.

Discussion

The system-parameter models used in this study are powerful tools for the qualitative and quantitative estimation of in situ soil properties. The recursive methods allow the behaviour of the soil system to be monitored throughout the excitation process and liquefaction, as shown in Figs 9–13. These figures show the changing characteristics of the soil system through time by plotting the spectral density evolution through time. This unique insight into the liquefaction process elucidates the reaction of the soil system to incoming energies, and eventual reduction in shear due to the rise in pore-water pressure. Confidence in the estimated models is increased by comparison with independent data from the E-W direction and the Elmore Ranch temblor. The estimates all converged on the values reported.

A question must be raised about the meaning of the damping values calculated in this study. A major problem is that the damping is being modelled as viscous damping, which is known to

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be a poor model but is the most amenable to calculation and the one most commonly used (Pandit, 1991). Even with a perfect technique, the damping values estimated by the model will not truly represent what is physically occurring in the soil. The values calculated, reasonable compared with other modal studies (e.g. Iemura, Yamada, Izuno, Iwasaki & Ohno, 1990) and many laboratory studies (Vucetic & Doby, 1991), are better seen as ‘effective’ damping coefficients, which capture all forms of frequency-dependent mechanical losses, than as an ‘intrinsic’ material property.

The damping estimates are often very sensitive to subtle changes in the modelling of the system, while the natural frequency estimates are more robust (Gersch, 1974). This is especially true for recursive estimates for systems that have very different damping values changing rapidly. In this case a limited number of data points enter directly into the calculation and variance is inversely proportional to the square root of the number of utilized data points.

The physical interpretation of the instantaneous mechanical values is also not immediately clear. The ARMA parameters recursively calculated at any given time define the filter needed to transform

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**Table 3. Summary of recursively estimated system parameters evolving through time during the Superstition Hills earthquake**

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>1st mode</th>
<th>2nd mode</th>
<th>3rd mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_r$ (Hz), $\zeta_r$</td>
<td>$p_r$</td>
<td>$f_r$ (Hz), $\zeta_r$</td>
</tr>
<tr>
<td>0.1</td>
<td>4.7, 10</td>
<td>0.33</td>
<td>—</td>
</tr>
<tr>
<td>2.4</td>
<td>3.9, 7</td>
<td>0.17</td>
<td>—</td>
</tr>
<tr>
<td>5.6</td>
<td>3.5, 90</td>
<td>−0.05</td>
<td>4.3, 8</td>
</tr>
<tr>
<td>8.6</td>
<td>2.4, 87</td>
<td>0.26</td>
<td>3.5, 15</td>
</tr>
</tbody>
</table>

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**Fig. 13. Evolutionary spectra for the coda (final 73 s) of the Superstition Hills earthquake: the surface represents the frequency domain model of the system through time, as the soil properties are changing with increasing pore pressure**
that time step of input data into the next output value. The mechanical characteristics are mathematically extracted from the filter based on a limiting set of assumptions, since an ‘instantaneous frequency’ has no physical meaning in itself. The ARX estimates are made with more input data points than the Kalman estimates, but the Kalman assumptions better model the changing system. There is no obvious choice between the ARX or Kalman model parameter estimates for the first two segments (0 to 12.8 s). A fundamental problem remains—at this point the geotechnical engineer does not really know what the ‘correct’ answer should be.

CONCLUSIONS

The Wildlife Site was subject to two large earthquakes on 24 November 1987. During the second, larger, earthquake (Superstition Hills), the site liquefied. The data collected are the only available records of buried and surface motions publicly available. In addition, pore-water pressure records were recorded at several depths during the temblor. The Elmore Ranch and Superstition Hills earthquakes were analysed from a seismological viewpoint to elucidate the site behaviour. It is shown that the Superstition Hills earthquake was actually three distinct subevents. The strong motion and pore-water pressure records were examined, and showed excellent correlation with the subevents. A significant rise in pore pressure that resulted in liquefaction of layer B occurred at the onset of subevent 3.

Both stationary and recursive system-parameter modelling methods were applied to the unique input–output data set from the two temblors. These unique results show weakening of the soils system as pore pressure increased, the recursive estimates being summarized in Table 3. While there are some important questions to be raised about the damping estimates, reasonable estimates were presented. Further work using system identification to estimate in situ soil properties is warranted.

REFERENCES

Ljung, L. (1979). Asymptotic behavior of the extended


