Optimizing embedded sensor network design for catchment-scale snow-depth estimation using LiDAR and machine learning


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Abstract.

We evaluate the skill of a methodology to identify optimal sensor placements for catchment-scale snow observatories using LiDAR and machine learning. Sampling locations that best represent catchment physiographic variables are identified with the Expectation Maximization algorithm for a Gaussian mixture model. A Gaussian Process is then used to model the snow depth in a 1 km$^2$ catchment surrounding the network, and additional sensors are placed to minimize the model uncertainty. We compare the skill of the snow depth model under the proposed placements to the model produced an existing sensor network at the Southern Sierra Critical Zone Observatory, where field surveys were used to place clusters of sensors to capture local variability in the physiographic features. Each model is validated with a 1 m$^2$ LiDAR-derived snow-depth raster from March, 2011. The proposed algorithm exhibits higher skill with fewer sensors (10 sensors, RMSE 7.4\%) than the existing network (23 sensors, RMSE 10.1\%). We assess the optimal long-term deployment strategy using hourly snow-depth data from the existing network and LiDAR-derived physiographic variables to quantify the effect of different water-year types on the independent variable correlations.
1. Introduction

Montane snowpack provides significant seasonal storage of freshwater for the Western United States and many other regions [Shafer et al., 1982; Bales et al., 2006]. The slow release of water during the spring and summer months ensures that multiple ecosystem and stakeholder needs are met throughout the year. Climate warming and changes in vegetation structure may significantly alter the timing and magnitude of storage and runoff in these watersheds [Goulden and Bales, 2014; Flanner et al., 2009]. These changes come at a time of increasing demand and potentially decreasing supply [Moser et al., 2009]. Water management will benefit from accurate, timely estimates of supply and runoff in order to efficiently allocate water. Existing regression-based hydrologic models, which use statistical relations from historical hydrographs to predict runoff and inform allocation decisions [Perkins et al., 2009; Rosenberg et al., 2011; Rango and Martinec, 1995] will have limited skill as conditions deviate from historical norms and are thus inadequate for water management.

To address these limitations, researchers have suggested blending remote sensing and in-situ measurements with distributed energy-balance models to better estimate storage and runoff [Guan et al., 2013]. These methods used well-developed remote-sensing [Painter et al., 2003; Rosenthal and Dozier, 1996; Dozier, 1989] and energy-balance tools [Marks et al., 1992; Link and Marks, 1999; Brubaker et al., 1996] to estimate snow and snowmelt processes across basins.

In-situ measurements for these methods are presently limited to snow pillows and snow courses, which largely sample flat, open terrain [Molotch and Bales, 2005], yet the distri-
bution of snow cover can vary considerably as a function of topographic features [Faria et al., 2000; Musselman et al., 2008]. To improve real-time snow distribution and melt estimates, and to inform decision support, in-situ measurements must be spatially re-
representative and capture gradients of physiographic variables such as elevation, slope, as-
pect, topologic concavity, and canopy coverage. Geostatistical techniques such as binary regression trees and kriging can then be used to estimate snow distribution across unin-
strumented regions [Balk and Elder, 2000; Erickson et al., 2005; Erxleben et al., 2002; Fassnacht et al., 2003; Harshburger et al., 2010].

To enhance the physiographic representativeness of existing snow-measurement net-
works, recent research has investigated the feasibility of using wireless-sensor networks to distribute spatially-dense snow measurements over a broader landscape (Figure 1). Prior work [Kerkez et al., 2012] and [Rice and Bales, 2010] showed that wireless-sensor networks can be configured to provide simultaneous measurements of snow-depth distribution, so-
lar forcing, and subsurface exchange across a 1-km$^2$ region. Based on these findings, 13 additional 1-km$^2$ scale networks are being deployed across the American River Basin to develop a real-time water information system [Kerkez et al., 2010].

Deploying multiple wireless-sensor networks at the basin scale using existing methods (such as random stratified sampling) is resource intensive and can result in sub-optimal networks. Extensive field surveys are presently required to identify representative sam-
pling regions for sensor nodes. These methods also do not account for heterogeneity in independent-variable correlations across sites, provide no metric for the number of sensor nodes required at each new site, and do not allow for the simultaneous optimization of sensor placements and repeater placements prior to deployment. These practices are un-
sustainable if wireless-sensor networks are going to see larger-scale adoption for real-time monitoring.

The research reported here investigates the feasibility of optimally structuring observatories prior to field deployment using high-resolution LiDAR data. The aims of the present study are to (i) quantify the skill of a machine-learning methodology, which identifies optimal placements in LiDAR-derived data (ii) assess how many sensors are needed to instrument the catchment using the proposed methodology (iii) compare the skill of the placements determined with the proposed methodology to an existing network in which placements were determined through field surveys, and (iv) assess the optimal long-term deployment strategy by quantifying the effect of different water-year types on the independent variable correlations.

2. Methods

2.1. Study area and data collection

The study site is located in the Southern Sierra Critical Zone Observatory (37° 4’ N, 119° 11’ W), within the rain-snow transition of the Sierra Nevada near Fresno, California. In WY2010, a 23-node wireless sensor network was installed, spanning a 1.5 km transect in a forested headwater catchment where the majority of annual precipitation falls as snow. The catchment covers a 1900-2100m elevation gradient, with slopes ranging from 0-40 degrees, and 76-99% canopy closures. Though predominately SW-facing, a range of aspects (0-359 degrees) exist within the catchment. Snow depth at each node is measured with an ultrasonic sensor. In order to inform a more complete estimate of the water balance, each node also measures temperature, relative humidity, soil moisture, and matric potential. For the present study, sensor locations in the existing wireless network were measured
with a Trimble GPS (10 cm horizontal accuracy). The snow depth was measured every 30 minutes using a Judd Communications ultrasonic snow depth sensor (1 cm accuracy). To account for beam divergence, sensors were oriented normal to the local slope. The raw data was smoothed using a daily average to remove noise.

Node locations in the existing network are clustered, so that multiple point measurements are used to estimate the mean snow depth. Particular attention was paid to capture the effect of canopy, with clusters of sensors measuring the drip-edge, under-canopy, and open regions at multiple locations in the catchment. Figure 1 shows the overall sensor distribution and typical network structure. Panel b shows a cluster of sensors in the network that were placed to capture drip-edge to open gradient in the NE region of the catchment. Other clusters in the network are designed to capture under-canopy effects as well as gradients of slope, aspect, and elevation.

Physiographic variables and a snow on/snow off raster were gathered from the NSF Open Topography database, opentopography.org [accessed 28 January 2016]. Elevation, slope, and aspect extracted from LiDAR data were processed in ArcMap 10.2. These data use 11.95 points per square meter to generate 1 m² DEM, canopy, and snow depth rasters. A comparison against ground-truth surveys conducted during the LiDAR over-flight showed 10 cm of vertical error in the snow-depth raster [Guo et al., 2010]. Elevation information was stored as point cloud in raw LiDAR data and the points of ground returns were gridded, averaged, and smoothed in order to create high-resolution digital elevation models (DEM). Slope and aspect were calculated from the gradient of the DEM in both longitudinal and latitudinal direction, which are the 2D convolutions of the elevation matrix and a sobel matrix or its transpose.
2.2. Identification of representative sampling locations

In the first step of the proposed methodology, we determine the distribution of sensors that is most-representative of the LiDAR-derived feature space. This is accomplished using a Gaussian mixture model, which assumes that a feature space (i.e., the combined LiDAR data from Section 2.1) is a product of a finite number of latent (unobserved) components (i.e., sensors). The sensor’s ability to observe each point in the feature space is represented using a multivariate normal distribution (Equation 1). This is the “kernel” function for the algorithm. The expected value of the normal distribution is the sensor’s location in the feature space. Multiple kernel functions are combined and weighted with mixing parameters (Equations 2 and 3). The combined ability of all sensors to observe all points in the feature space is represented using a likelihood function (Equation 4).

Formally, the Gaussian mixture model is a linear superposition of $D$-dimensional multivariate normal kernels, $N$, with expected value, $\mu$ and covariance $\Sigma$ applied to data, $x$:

$$N(x|\mu, \Sigma) = \frac{1}{(2\pi)^{D/2} |\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(x - \mu)^T \Sigma^{-1} (x - \mu)\right\}$$ (1)

Superposition, with mixing parameters, $\pi_k$:

$$p(x) = \sum_{k=1}^{K} \pi_k N(x|\mu_k, \Sigma_k)$$ (2)

Subject to:

$$\sum_{k=1}^{K} \pi_k = 1$$ (3)

The complete log-likelihood function is given by:
\[
\ln p(D|\pi, \mu, \Sigma) = \sum_{n=1}^{N} \ln \left\{ \sum_{k=1}^{K} \pi_k \mathcal{N}(x_n|\mu_k, \Sigma_k) \right\}
\] (4)

Algorithms exist to solve for latent components using both a Bayesian and Expectation Maximization (EM) framework \cite{McLachlan2004}. The method we present uses the EM algorithm \cite{Pedregosa2011}. This is an iterative process in which the algorithm tries to recover the most likely parameter estimates for the mixture of multivariate kernels to explain the data. We used a spherical covariance function and updated the model weights, covariance, and means with each iteration. Once the maximization step no longer increases the log-likelihood, the process terminates and the optimal sensor locations have been found. Like many gradient-based optimization methods, EM converges to local minima. Therefore we initialized the algorithm with K-means clustering, a vector quantization method which divides the feature space into \( K \) Voronoi partitions. The centroid of each partition is taken to be the initial estimate of the expected values for the Gaussian mixture model.

The converged Gaussian mixture model is shown in Figure 4. Optimal sensor placements (the expected values of the latent gaussians) are shown as red markers. The likelihood function (quantifying how well the space of independent variables is observed under the sensor current configuration), is shown as contour lines. Points that are well-observed are shown in blue; poorly-observed points are shown in red. As it is not possible to show the full six-dimensional feature space, Figure 4 is a two-dimensional projection of the output. The spatial distribution of sensors corresponding to the projection in Figure 4 is shown in Figure 8.
2.3. Snow depth model

In the second step, we use snow-depth measurements at the locations proposed by the Gaussian mixture model to estimate the distribution of snow depth across the catchment using a Gaussian Process (Equation 5). All of the independent physiographic variables (slope, aspect, elevation, and canopy) are used in the estimation process (i.e. universal co-kriging). The Gaussian Process combines the point measurements from each sensor station and the four LiDAR-derived physiographic variables to estimate the mean snow depth, \( m \), using a covariance function \( K \). The resolution of the snow-depth model is 1 m\(^2\).

\[
X \sim GP(m, K) \quad (5)
\]

We used a squared exponential covariance function (Equation 6), which depends on four variables: \( x \) and \( x' \) are two points in the domain, \( d \) is the distance between them, and \( l \) is the characteristic length scale. The spatial autocorrelation in the model is controlled by the \( l \) parameter. We assume the algorithm has no prior information about this parameter; it must be estimated only from the point measurements at each sensor. The error at each measurement (10 cm based on the LiDAR error) is quantified using the nugget effect in Equation 7, where \( y_i \) is the measured snow depth at point \( i \), and \( \sigma_i \) is the measurement variance at \( y_i \).

\[
K_{se}(x, x') = \exp\left(-\frac{|d|^2}{2l^2}\right) \quad (6)
\]

\[
\text{nugget}_i = \left[\frac{\sigma_i}{y_i}\right]^2 \quad (7)
\]
2.4. Supervised updates

In addition to estimating the spatial mean of the snow depth, the Gaussian process estimates the distribution of model uncertainty as a function of the independent variable weights and estimated autocorrelation. This provides a basis for placing additional sensors with the aim of minimizing uncertainty throughout the catchment. The additional placements are considered “supervised” updates because they rely on observations of the independent variable (i.e. the estimated spatial autocorrelation of snow depth) to determine optimal locations. They are distinct from the placements in Section 2.2, which are “unsupervised” in the sense that no observations of the dependent variable are used in their determination. Sensors placed in regions of high uncertainty will reduce the uncertainty throughout the domain at points with similar combinations of physiographic features.

2.5. Model evaluation and optimal number of sensors

The error in the snow-depth model was determined by differencing the predicted snow depth under each scenario from the LiDAR-derived snow-depth raster (Figure 2). We quantified the skill of the snow depth-model under each placement scenario using two standard metrics: RMSE (Equation 8) and Bias (Equation 9) In all equations, \( n \) is the number of points in the model, \( x_i \) is the model prediction at point \( i \), and \( y_i \) is the true snow depth.

\[
\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - y_i)^2}
\]  
(8)
\[ \text{Bias} = \sum_{i=1}^{n} (x_i - y_i) \]

We investigated how many sensors are needed in the proposed methodology by examining the error in the snow-depth model in a range of placement scenarios. In the first set of scenarios, we began the unsupervised placements with 2 sensors, increasing to 23 sensors. The optimal number of sensors was taken to be when marginal improvement in RMSE was less than 1%. With the optimal number of unsupervised placements determined, we examined the uncertainty in the Gaussian Process snow-depth estimate. We extracted a feature space of the regions corresponding to the highest 10% of model uncertainty. The optimal placements were determined by running the Gaussian mixture model again within the high-uncertainty regions. This process determines the most physiographically diverse placements in the high-uncertainty regions.

Finally, we compared the optimal number of sensors in the algorithm to an equivalent number of randomly chosen, but spatially-distributed sensors. We evaluated the snow-depth model under 100 configurations to determine the expected skill of the random placements.

### 2.6. Long-term controls on snowpack

We assessed the long-term controls on snow depth in the catchment by combining the LiDAR physiographic data with daily-average snowpack data from the existing wireless sensor network. On each day during WY2011-WY2013 we computed the coefficient of determination (Pearson correlation coefficient, Equation 10) for each independent variable in the model.
\[ R = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}} \] (10)

3. Results

3.1. Unsupervised placements

We found that the autocorrelation parameters of the snow-depth model are not reliably recovered by the algorithm if fewer than six sensors are placed in the catchment in the unsupervised process. The snow depth estimated from four sensors determined using the unsupervised process is shown in Figure 7. Adding sensors in the unsupervised step improved the skill of the algorithm, but only modestly. With 10 sensors the RMSE in the snow-depth model is 8.6%, bias is 17 cm, and \( R^2 \) is 0.53. At 23 sensors the RMSE was only reduced to 8.48%. Between 10-23 sensors the RMSE ranged from 8.5-9.3%, depending on the particular placement determined by the algorithm, but was never lower than 8.48%.

With six sensors, the distribution of sensors shown in the converged Gaussian mixture model appears to cover the normalized feature space (Figure 4). The cumulative distribution functions from Figure 5 show that the distribution of sensor nodes closely approximates the distribution of LiDAR data and samples across the full range of each variable.

We compared the results of the Gaussian mixture model algorithm to placements done by field survey (Figure 5). The Gaussian mixture model results are generally comparable to field placements, and in some cases they are significantly better. For example, lower values of aspect (i.e., N to NE-facing points) are under-sampled by the field survey at the SSCZO, as are high values of elevation.
3.2. Supervised process

Additional sensors placed in the supervised process resulted in greater improvements to the snow-depth model. The uncertainty in the snow-depth model using the unsupervised placements is shown in the left-hand panel of Figure 6. The magnitude of the spatial distribution corresponds to the width of the 95% confidence interval in the Gaussian Process: it is a function of the local distribution of independent variables, the variable weights, and the estimated autocorrelation.

Using six sensors the uncertainty in the snow-depth model showed distinct areas of high uncertainty in the NW and SW regions of the catchment. In both regions, there is a large amount of topographic complexity (large variations in slope, aspect, and vegetation). We extracted the distribution of independent variables corresponding to these regions and used the Gaussian mixture model to find representative placements for the additional sensors. The greatest reduction in RMSE was associated with placing four additional sensors in these high-uncertainty regions (red “+” markers in the left hand-panel). The right-hand panel shows the updated uncertainty: it is reduced throughout the catchment. The reduction in uncertainty in each cell depends on how similar its physiographic features are to the additional placements’ features. In Figure 8, the middle panels show the output of the snow-depth model with the updated placements. Throughout the region, the estimate is moderately improved: the mean error is reduced to -8 cm, the RMSE is 7.4 %, and $R^2$ is 0.59.

3.3. Random placements

In the 100-sample evaluation of randomized placements of 10 spatially-distributed sensors, the average RMSE was 10.6 %. The distribution was heavy-tailed: most of the
results were clustered between 8-12 % RMSE, but there were 10 outcomes with greater than 20 % RMSE. A configuration corresponding to one of the average runs is shown in the middle panel of Figure 8. The spatial distribution is similar to the proposed placements, however the output of snow-depth model reveals a slight over-weighting of the slope variable in the regression. Although there is only one region of high error (the SW corner), the model generally produces an over-estimate of the true snow depth (Figure 9): the mean error is -20 cm.

3.4. Existing network

The snow depth model estimated from the existing placements has higher error (RMSE 10.1 %, mean 25 cm, and $R^2$ is 0.49) than the estimates from the machine learning method, but lower than than the randomized placements. The skill of the snow-depth model is high (less than 15 cm error) near the sensor clusters. However, the error is very high (greater than 1 m error) in the NW and SW region, where the snow-depth model over-estimates the true snow depth in the topographically-complex regions. This results in a more heavy-tailed error distribution than the proposed or random placements, and an overall overestimate of the true snow depth within the catchment.

3.5. Long-term analysis

In WY2011-WY2013, the correlation coefficients of slope, aspect, elevation and canopy show consistent inter-annual patterns. Canopy is the strongest predictor of snow depth. It is negatively correlated (i.e. there is lower snow depth in denser canopy), and the magnitude of the coefficient of determination is consistently 0.8 when snow is present at all nodes. Elevation initially shows zero or slight positive correlation, but becomes
negatively correlated during the ablation process (i.e. snow melts at higher elevations first). Slope is generally negatively correlated (i.e. steep slopes have less snow depth), but the relationship becomes weaker as the snow melts. Aspect is consistently positively correlated, but the relationship appears to be strongest during first snowfall. During the drought years (WY2012 and WY2013), the correlation coefficients show large fluctuations when the snowpack is extremely low (< 10 cm).

4. Discussion

Although individual point measurements of snow depth are poor estimators of the local mean, our results indicate that a limited combination of representative placements can be used to estimate the catchment-scale snow cover. Ten placements in strategic locations produced a better catchment-scale estimate than 23 placements in the existing network, which were clustered in order to capture the local mean. While the existing network aims to sample localized gradients in physiographic variables, it appears that these do not provide the measurements needed to estimate the catchment-scale distribution. The skill of the existing network is high in the vicinity of the sensor clusters, but low in the NW and SW regions of the catchment, which feature a large amount of topographic complexity. When designing catchment-scale snow observatories, it is likely better to capture physiographically unique regions (i.e. with a particular combination of physiographic variables), than to sample localized gradients.

We found that the network optimization can be accomplished in an automated process in which a Gaussian mixture model (Figure 4) determines sensor placements with representative combinations of physiographic variables, and measurements at the proposed locations are used to estimate the uncertainty throughout the catchment. This second
step is shown in Figure 6 where the NW and SW regions are identified as having high uncertainty, and a number of additional sensors were placed.

Prior studies [Rice and Bales, 2010] found that most of the benefit is derived from the first 4-5 additional sensors when using a binary regression tree to distribute snow cover. While we found that 5-6 sensors can be placed in an unsupervised manner in order to estimate the spatial distribution, we found that with fewer than 4 sensors, the autocorrelation in the snow-depth model is not adequately estimated from the network alone. This limitation could be addressed by bounding the autocorrelation parameter in the snow-depth model, but this requires prior knowledge of the typical local catchment-scale snow properties.

None of the algorithms captured the increasing, S-facing gradient of snow depth across the meadow, which was likely a result of wind redistribution. This effect has been observed in a number of prior studies [Winstral and Marks, 2002; Molotch and Bales, 2005], and directional variables have been suggested to account for directional redistribution of snow (e.g. Molotch and Bales [2005]). Given the dense canopy throughout the catchment in the present study, we did not include a directional variable in the feature space. In catchment with less dense distributions of canopy, it would likely be necessary. Care should be taken in mixed open/forested regions that the directional bias measured in open regions is not translated to the regions with dense canopy.

While the LiDAR snow-depth raster used in the present study provides a high-resolution ground-truth for each model, time-series data are needed to assess the long-term deployment strategy. The long-term analysis using the existing WSN data indicates that placements are consistent across water years, implying that the distribution of sensors chosen by
the algorithm is appropriate across water-year types. This consistency may not be present in regions with different physiographic variables, and future studies could consider how to adapt the algorithm to regions in which the relative controls of the independent variables might change. Also, though inter-annual controls are consist in the present study, there is some variability within the year (e.x. the declining effect of slope in WY2011 (Figure 10)).

The distribution of sensors should be tuned to which part of the water year is prioritized.

There are practical considerations in observatory design that are not considered in the present study. Regions of the catchment may be inaccessible due to terrain attributes and other access constrains. This can be addressed by defining a set of inaccessible placements in the feature space from Figure 4. If the optimal sensor location lands on an inaccessible grid element, a search to the nearest viable point in the feature space will be output as the optimal point. The proposed algorithm also requires a greater spatial distribution than the clustering approach. Recent field deployments have indicated that 1 km-scale wireless sensor networks can be deployed using existing hardware in a variety of terrains. However if the spatial extent is limited, the spatial coordinates could be removed from the unsupervised step, and an algorithm could determine the most-representative, spatially-proximate distribution of nodes.

It should be noted that the “supervised updating” step in the present study would require two field deployments: one to gather data to estimate the distribution of uncertainty throughout the catchment, and a second to add sensors in high-uncertainty regions. In practice, the marginal gain from the supervised updates may not be worth the labor required to install additional sensors, though in our analysis it appears that supervised placements outperform unsupervised placements.
Finally, the true optimal number of nodes per site should be determined by considering the marginal value of the improved information. Combined with an understating of the marginal cost of each additional placement, this would determine an optimal number of sensor nodes by comparing marginal cost and value functions. This approach would capture site-specific value and marginal costs of the sensor network, which vary from site to site. For example, a site directly above a hydropower plant may have a higher marginal value of information.

5. Conclusion

There are four conclusions relevant to the aims discussed in Section 1. First, the two-step methodology to identify representative sampling locations resulted in a snow-depth model with a mean error of -6 cm and RMSE of 7.4%. Second, with fewer than five sensors, the autocorrelation parameters in the algorithm were not accurately estimated by the sensor placements. In the unsupervised step, there was limited improvement in the RMSE when placing more than six sensors. Third, the snow depth model using the placements proposed by the algorithm was higher than using the clustered placements in the existing network. Fourth, the long-term controls on snow depth in the existing network were not substantially altered by different water-year types.

While the research reported here indicates that structuring catchment-scale observatories prior to field deployment using remotely-sensed data may be a feasible alternative to conducting field surveys, further research is needed to quantify the skill of the algorithm over longer time spans, and in different environments. In high-elevation terrains, the relative skill of the independent variables is likely weighted towards terrain attributes (slope, aspect, wind-redistribution etc.) rather than the distribution of canopy. It would be best
to evaluate the long-term skill of the algorithm in different environments using multiple years of LiDAR data.

It would be informative to use the proposed methodology to determine the relationship between the number of sensors required to instrument the region and the topographic complexity of the catchment. The algorithm could be run in multiple regions with varying degrees of topographic complexity, and an empirical relationship may emerge which could serve as an objective guideline when establishing new observatories. Finally, it would be useful to investigate how increased information from these networks translates to value for downstream users. This would provide a more objective metric for the marginal value of information, which could be compared to the marginal cost of network establishment to determine the best locations for new networks.

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Figure 1. An existing wireless sensor network at the Southern Sierra Critical Zone Observatory. Clusters of ultrasonic point measurements of snow depth are distributed across a 1.5 km transect in a 1 km² catchment. Data from each cluster is relayed every 15 minutes through a network of wireless elements to a base station with a real-time data uplink.
Figure 2. Snow depth in the 1 km$^2$ catchment is used to validate the model output from each sensor configuration (shown in Figure 8). The resolution of the snow-depth raster is 1 m$^2$ with 0.1 m vertical error. The data are derived from the snow on/snow off LiDAR data from Guo et al. [2010].
Figure 3. The distribution of independent variables in the 1 km² catchment is used to determine optimal sensor locations. Slope, aspect, elevation, and canopy are derived from the 1 m² LiDAR raster [Guo et al., 2010]. Each panel is a two-dimensional projection of the six-dimensional feature space. Figure 4 illustrates how representative sensor placements are determined in the feature space.
Figure 4. Representative sensor locations are determined using a Gaussian mixture model in the space of independent variables from Figure 3. The feature space is normalized to ensure each variable is evenly weighted. Optimal sensor locations (indicated by red points) are the expected values of the latent Gaussians. The likelihood function (contour lines) quantifies how well each point in the LiDAR-derived feature space is observed given the locations and covariance of each sensor (see Equation 4). The optimal parameters (expected value and covariance) for the model are determined using the EM algorithm [McLachlan and Peel, 2004].
Figure 5. Current node placements (determined using field surveys) compared to Gaussian mixture model placements at the SSCZO. Both methods provide a representative sampling of each physiographic variable except aspect, which is under-sampled by the existing sensor clusters. Elevation above 1900 m.
Figure 6. The supervised updating process. In the left-hand panel, snow depth measurements are taken at locations selected by the Gaussian mixture model (Figure 4). The snow depth is then estimated throughout the catchment with a Gaussian process. Additional sensors (indicated by red + markers) are then placed in the regions of highest uncertainty in the model: this reduces the uncertainty throughout the catchment (right-hand panel).
Figure 7. An insufficient number of sensors in the unsupervised process results in poor estimation of the regression weights and spatial autocorrelation in the snow-depth model. We require that these parameters be estimated solely from the measurements at each sensor.
Figure 8. Predicted snow depth (left-hand panels) and error (right-hand panels) using three sensor configurations: proposed (top), random (middle), expert (bottom). The resolution of the snow-depth model is 1 m$^2$. The error of each model is computed relative to the LiDAR snow-depth raster in Figure 2. The distribution of errors is compared in Figure 9.
Figure 9. The distribution of error in the snow-depth model under each sensor configuration from Figure 8. The proposed placements have the least bias and lowest error (mean: -0.08 m, RMSE: 7.4%), compared to expert (mean: 0.25 m, RMSE: 10.1%) and random (mean: -0.20 m, RMSE: 10.6%).
Figure 10. Effect of water year type on independent variables used to estimate snow depth. Snow depth data (top panel) is measured at 20 sensors in the existing wireless sensor network. Pearson coefficients (lower panel) are derived at each point from the LiDAR physiographic variables.