Body waves recorded inside an elastic half-space by an embedded, wideband velocity sensor

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This paper presents a unique embeddible acoustic emission sensor. Comparison with theoretically calculated waveforms for the embedded sensor, surface step force Lamb’s problem prove the sensor to be an accurate transducer of particle velocity, with sensitivity of 2.34 V output per mm/s. Calibration as a surface sensor by the National Institute of Standards and Technology (NIST) show the sensor to be an accurate transducer of surface displacement with a sensitivity of 2.8 V/nm. The paper presents details about the design and construction of the sensor as well as calibration and verification. Unique design elements include the use of a lead-alloy backing masses, soft elastomer shear-spring isolation and mounting, and embedment. The sensor is based on the NIST conical lead zirconate-titinate (PZT) element and has a finished length of 38 mm and a diameter of 16 mm. The sensor is robust enough to work under 1 MPa of brine pressure. © 1998 Acoustical Society of America. [S0001-4966(98)01409-X]

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INTRODUCTION

Traditionally, acoustic emission (AE) studies are conducted by placing an array of sensors on the surface of the specimen being tested. Unfortunately the physical constraints of many practical testing geometries preclude surface placement of sensors. A simple solution to this problem is to place the sensors inside the body to be monitored, but to date no such sensors have been available. This paper introduces a unique, embeddible, high-fidelity particle velocity transducer, including design parameters and National Institute of Standards and Technology (NIST) calibration. The design yields sensitivity equal to commercial resonant devices while maintaining the high fidelity of the NIST displacement sensors. The embeddible high-fidelity sensor discussed here for the first time allows experimental verification of Lamb’s problem of body motions due to a surface excitation. Validation of the performance of the sensors is provided by a parametric study of recorded and theoretical waveforms for Lamb’s problem, buried transducer, surface source.

Using embedded sensors has several advantages over surface mounted sensors, including: (i) mode conversion is avoided so the measured waveforms are simpler and easier to interpret; (ii) a greater length of signal can be recorded before being contaminated by surface reflections/mode conversions; (iii) the sensor(s) can be located closer to the area of crack growth; (iv) embedded sensors may be less susceptible to accidental movement or damage; (v) embedded sensors are the only option when free surfaces are not accessible (Glaser, 1991).

Most acoustic emission studies use resonant sensors because they provide greater sensitivity near the resonance frequency. Unfortunately, resonant devices lack the bandwidth needed to analyze incoming waveforms with the rigor with which seismologists study earthquake waveforms (e.g., Aki and Richards, 1980). Resonant sensors provide a reasonably accurate estimate of the AE time of arrival, but beyond the direction of first motion the received signal is more a function of the sensor than of the kinematics at the crack tip (e.g., Pao, 1978; Hamstad and Fortunko, 1995). An acoustic emission study using resonant sensors “is empirical, relies on correlations, and suffers from a very thin causal science basis” (Eitzen et al., 1981).

By using a wideband sensor having a flat frequency response over several octaves, the recorded signal is equivalent to the actual kinetics at the receiver location, enabling researchers to determine not only the time of first arrival, but also the arrival times of the P wave, S wave, and boundary reflections, as well as their relative magnitudes (e.g., Glaser and Nelson, 1992b; Kishi, 1985). It has been shown that by using a high-fidelity sensor, the inverse problem of determining the source function from remote measurements can be achieved (e.g., Eisenblatter, 1980; Eitzen et al., 1981; Kim and Sachse, 1986). Full waveform signals also allow the use of forward modeling to evaluate source kinematics (e.g., Aizawa et al., 1987).

I. BACKGROUND AND THEORY

A. Lamb’s problem

Wave propagation problems involving an elastic half-space have come to be known as Lamb’s problems due to the early work in the field by Horace Lamb (1904). Lamb looked
at surface response due to a surface excitation by integrating Rayleigh’s (1887) discovery of the surface wave. Both researchers were attempting to understand surface response to earthquake excitation. Lamb noted that the $P$- and $S$-wave history (minor tremors) was a function of the time derivative of the source function while the surface wave (major tremor) history was a direct function of the source kinematics (Lamb, 1904).

The first partial solution to the more complicated half-space problem, internal motions due to surface excitation, was published by Miller and Pursey (1954). Miller and Pursey derived integral representations of the internal motions due to simple harmonic motions normal to the surface of the half-space. Pekeris provided a full solution for the motion of the half-space surface due to a buried Heaviside pulse in terms of a single integral (Pekeris and Lifson, 1957). Five years later Cherry (1962) published a solution for body and surface wave propagation due to a horizontal motion on the surface of the half-space.

Johnson (1974) presented a complete solution for the three-dimensional Lamb’s problem in the form of a Green’s function and spacial derivatives for an elastic, uniform half-space. Ohtsu and Ono (1984) published a computer program simplifying Johnson’s numerical procedure for the surface receiver–interior source case. Using ray theory, Pao and Gajewski (1977) provided general solutions for transient waves generated by a multitude of sources for layered solids, with more detailed solutions derived by Mal (e.g., Mal et al., 1982). Application of these solutions to decoding acoustic emission source events include work by Wadley (e.g., Wadley and Hill, 1983) and Michaels (Michaels et al., 1981).

### B. Solution of Lamb’s problem

Using the viscoelastic propagator method introduced by Kennett (1983), synthetic seismograms were generated for a uniform half-space having the elastic properties listed in Table I. The attenuation properties of the rock half-space were modeled with the quality factors $Q_p = Q_s = 50$, and the constant $Q$ model of Kjartansson (1979) was used for the dispersion relationship. These synthetic seismograms are an appropriate solution for our problem: a unit step function in force acting on the free surface, a receiver that records vertical motion at a depth of 152 mm within the half-space, and at a range of horizontal offsets from the source position. The calculations were performed for a buried source and surface receiver and then reciprocity was used to obtain the results for the experimental geometry.

### II. DESIGN OF THE EMBEDDED HIGH-FIDELITY ACOUSTIC EMISSION SENSOR

A cutaway schematic of the sensor as built is shown in Fig. 1. The following sections explain how the sensor is constructed, with further details in Weiss and Glaser (1998).

#### A. Sensor element

The embedded sensor uses a 2.5-mm-tall truncated cone of PZT-5a (a lead-zirconate-titanate composition), with an aperture diameter of 1.5 mm and a base diameter of 6.5 mm. This geometric design of piezoelectric cone was developed by the National Institute of Standards and Technology (NIST) for use in wideband acoustic emission sensors (e.g., Eitzen et al., 1981; Proctor, 1982a, b). The conical design eliminates the aperture effect by keeping the contact area small, is sensitive to only one parameter (motion in line with the cone axis), and reduces the degeneracies of the normal modes of the usual piezoelectric disk element (Proctor, 1982b; Greenspan, 1987). The element has a high piezoelectric coupling constant and a mechanical stiffness estimated to be $1.1 \times 10^8 \text{ N/m}$ at a minimum (Hamstad and Fortunko, 1995).

#### B. Electronics system

Only a small fraction of the energy released from an acoustic emission event ever reaches a sensor. Furthermore, a piezoelectric sensor converts only a fraction of the received energy into an electrical signal. Unless this signal is amplified at the crystal, it will be corrupted by electromagnetic noise and capacitive loading from the cable between the sen-
cables are approximately 95 pF/m, while the conical element
ded sensor, originally developed by Shiwa et al.

C. Backing mass

Previous research (Proctor, 1982a, 1986) has shown that
the geometry of the backing mass is the single most im-
portant element of wideband acoustic emission sensor design. In
general, a larger backing mass will have fewer resonances
and will give a more satisfactory performance at lower fre-
quencies. A unique problem was presented since the embed-
ded sensor was not supposed to affect the passing wavetrain.
Thus the backing mass has been carefully designed to be as
compact as possible, while reducing resonances and maxi-
mizing the total mass. The feasible choices were tungsten
and lead, lead having never before used as an AE sensor
backing material. Of the two, lead has a much lower $Q$
thereby minimizing possible resonance. Lead proved difficult
to machine and solder so a lead-based babbitt metal
is relatively insensitive to hydrostatic pressure. Increasing surrounding fluid pressure actually causes
the rubber to seal more tightly against water infiltration,
whereas pressure proofing a rigid casing would be much
more problematic. The surfaces of the sensor internal com-
ponents are coated with a primer to provide strong adhesion
with the rubber, further preventing fluid infiltration. Unfor-
unately the rubber did not provide acoustic isolation be-
tween the host specimen and the sensor internals, resulting in
coupling between the backing mass/PZT tip and specimen.

D. Casing and acoustic isolation

A unique sensor casing was designed so that the sensor
would accommodate strain and deformation of the host ma-
terial, and operate in a brine pore fluid environment under
pressure approaching 1 MPa. This has been achieved by
pouring a soft (Shore-A hardness = 9), moldable room tem-
perature vulcanizing (RTV) rubber between the tubular brass
sensor casing and the internals. The RTV rubber provides
longitudinal compliance between the sensor casing and inter-
als to maintain intimate contact between sensor tip and
specimen (i.e., shear spring). The casing constrains the rub-
ger from expanding laterally when a coupling force is ap-
plied to the rear of the sensor, allowing the sensor to be
placed in a hole just slightly larger than the casing without
significant traction forces. RTV rubber has a high Poisson’s
ratio (0.498), and is relatively insensitive to hydrostatic pres-
sure. Increasing surrounding fluid pressure actually causes
the rubber to seal more tightly against water infiltration,
whereas pressure proofing a rigid casing would be much
more problematic. The surfaces of the sensor internal com-
ponents are coated with a primer to provide strong adhesion
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tween the host specimen and the sensor internals, resulting in
coupling between the backing mass/PZT tip and specimen.

III. EXPERIMENTAL METHODS

A. Sensor calibration methods

Laboratory development of the sensor was guided by
frequent calibration. Calibrations were performed on a large
50-mm-thick steel plate, with the signal being measured
and the top surface, 76 mm from the source, also input on the
plate top (Hsu and Breckenridge, 1981). Both capillary and
3.0-mm mechanical pencil lead breaks were used as calibra-
tion sources per standard AE practice (e.g., Breckenridge
et al., 1990). Although the pencil lead source only ap-
proaches a true step impulse, it is convenient to use, very
repeatable, and makes for easy comparison with the work of
most AE researchers who use this source exclusively. Figure
3 shows a comparison of impulse response of the embeddible
sensor to the theoretical infinite plate solution of the same
geometry and loading (Hsu, 1985). The sensor response dif-
fers little from the theoretical, with most of the difference
due to the lack of 0–12 kHz energy that the piezoelectric
device cannot sense. The functional difference is the over-
shoot on the initial spike arrival rebound shown by the sen-
or. During design and assembly the embeddible sensor re-
response was also directly compared to the output of a large, brass-backed conical sensor, built and calibrated to NIST Standard Reference Material (SRM) specifications (Glaser and Nelson, 1992a). Final absolute surface calibration of the embeddible sensor was done by the Ultrasonics Standards Group, National Institute of Standards and Technology, Gaithersburg, MD (Fick, 1996). Figure 4 shows the actual displacement spike measured by the NIST capacitive standard transducer compared with the response of our sensor. Here too, as seen in Fig. 3, there is overshoot on the rebound for the embeddible sensor. The results of NIST calibration of the surface-mounted embeddible sensor is shown in Fig. 5 as frequency magnitude and phase in terms of absolute displacement compared to the frequency response of the NIST SRM conical sensor. Note that the response of our sensor is virtually flat from 12 to 960 kHz, with an absolute sensitivity between 28 and 30 V output per nm displacement. For the phase response, the statistical uncertainty of the measurement is greater than the induced phase shift. This calibration shows that the sensor gives an accurate time measure of very small surface displacements over a broad bandwidth.

B. Experimental verification of Lamb’s problem

In order to verify the embedded behavior of the sensor, a set of experiments was run to measure the in situ body wave response for the buried sensor–surface force Lamb’s problem. A sketch of the experimental geometry is shown in Fig. 6. A large (0.33-m cube) dolostone block was used as the “half-space.” The block is homogeneous and isotropic for the scale of the test (Nelson and Glaser, 1992), and material properties are given in Table I.

The sensor itself was cemented into a 20-mm-diameter borehole 152 mm beneath the horizontal surface upon which the source acted. A hole was bored into the bottom of the block with a flat-end diamond tool, and the bottom hand-ground flat. The sensor is then inserted tip-down into the hole, with the tip resting on the bottom, and the annulus filled with very low viscosity epoxy. Downward pressure on the casing before and during curing results in 1.5-mm axial deformation of the casing relative to the sensor tip. This displacement forces the sensor tip into intimate contact with the rock by the shear-spring action of the RTV rubber in the annular space between the brass casing tube and the sensing element. The beauty of our design is that the shear spring insures that intimate contact is maintained after cure shrinkage of the epoxy, and specimen deformation. The entire open volume between the sensor and rock is filled so that there is no free surface near the sensing element.

The step source used for this experiment was a glass
capillary break rather than pencil lead since the capillary source imparts significantly more force, and better approaches a true step force impulse. For the upper frequency bound of interest to us, 1 MHz, the capillary source yields a near-perfect step drop normal to the horizontal surface (Breckenridge et al., 1990). NIST typically uses a 0.12-mm o.d. capillary source, yielding a 4-N step force with a 0.25-μs rise time. For our experiment the source pulse must penetrate through more than 215 mm of attenuative and dispersive rock and still have an amplitude well above the noise floor. Therefore 0.5-mm o.d. capillaries were used, yielding a 42-N force, and an estimated rise time of 1.1 μs.

The first signal was recorded with the step-source in the epicentral position on top of the rock block, 152 mm above the sensor face. The 42-N step force was input on the surface at six successive 25-mm horizontal steps from epicenter (Fig. 6). Normalizing linear dimensions by the 152-mm embedment depth, \( D \), the step source was input at 0.167\( D \), 0.333\( D \), 0.5\( D \), 0.667\( D \), 0.833\( D \), and \( D \). All experimental signals were recorded at 14-bit resolution and a digitization rate of 10 mega samples/second. Only the initial portion of each event waveform was used for analysis, from the initial arrival of event energy with the \( P \) wave to just after the arrival of the \( S \) wave. The use of only the initial, uncontaminated segment of the waveform insured that the analysis would be free from the effects of sample size and shape, and modal analysis would not need to be undertaken. The signals analyzed were effectively from an infinite half-space.

**IV. VERIFICATION OF SENSOR PERFORMANCE**

**A. Epicentral results, experiment and theory**

A rigorous verification process was undertaken to calibrate the embedded sensor to the theoretical particle velocity time histories for the given geometry. The epicentral geometry was used as the fundamental calibration case. The sensor response to a 42-N step force (capillary break) input normal to the top surface directly above the center-line of the embedded sensor is shown in Fig. 7, with output given as absolute particle velocity time history. For comparison, the theoretical particle velocity time history for this scenario is also shown in Fig. 7. The segments of signals shown covers the period from just before the \( P \)-wave arrival to just after the \( S \)-wave arrival.

Numerous experiments had shown that the dispersive rock effectively filters out frequencies above 450 kHz for travel paths much over 50 mm (Nelson and Glaser, 1992), so the only processing of the theoretical waveform was 450-kHz low-pass filtering (single-pole bidirectional Butterworth) that mimics the dispersive attenuation of the rock half-space. The experimental time history is presented without signal processing, scaled to theoretical peak first-arrival \( P \)-wave velocity.

**B. Results from various geometries, experiment and theory**

The embedded sensor is designed to be sensitive strictly to motion in-line with the sensor axis. To verify the design, step forces were input on the horizontal top of the test block at various off-epicentral locations (hence azimuths) relative to the sensor axis (Fig. 6). Given a point source–point receiver, the radiation pattern for both \( S \) and \( P \)-wave motions will have significant energy normal to the plane being monitored. The experimental results were then compared to the theoretical waveforms for the same off-epicentral locations. The comparisons are shown in Fig. 8(a)–(f). The capillary source step force results for offsets of 25 to 152 mm are shown for verification.

**C. Evolution of the head wave**

In an elastic half-space there exists (for some geometries) a refracted shear-wave referred to as a head wave (e.g., Pao and Gajewski, 1977; Cerveny and Ravindra, 1971). The high fidelity of our sensors was further tested by recording waveforms at source offsets of 107, 112, 117, and 122 mm. Theoretical velocity time histories were also calculated for these geometries. Results of theory and experiment are compared in Fig. 9. Again, the figure shows the waveform from pre-\( P \)-wave to post-\( S \)-wave arrival, before mechanical resonances of the specimen and source contaminate the waveform.

**V. DISCUSSION OF RESULTS**

**A. Discussion of sensor epicentral performance and calibration from theory**

Figure 7 presents the epicentral particle motion time history from the initial arrival of the longitudinal wave (\( P \) wave) to just after the arrival of the shear wave (\( S \) wave). The rise time of the initial arrival of \( P \)-wave energy as measured by the sensor matches the theoretical solution for particle velocity exactly. The behavior at the arrival of the \( S \) wave is also in congruence, as is the gradual downward slope between the \( P \)- and \( S \)-wave arrivals.

After extensive theoretical modeling and comparison to the recorded signals, it was decided that the sensor measures particle velocity when embedded, in contrast to its being a...
particle displacement transducer when surface mounted. When surface mounted, the sensor casing is physically isolated from the half-space. When embedded, it was initially assumed that the soft RTV rubber potting would serve to isolate the active sensor elements from the host motions. However, it was found that silicon rubber has a glass transition frequency at about 10 kHz. For example, the Young’s modulus of a soft RTV-141 rubber increases almost three orders of magnitude in the 10-kHz frequency range resulting in a high-frequency stiffness of 5 GPa (Bosc and Mauguen, 1990). We are currently evaluating a urethane elastomer which is quite acoustically isolating at high frequencies (Selfridge, 1998).

The congruence between the measured and calculated particle velocity time histories gives credence to the calibration of the sensor from the Kennett propagator theoretical solution, which accounts for the source kinematics and geometry of the capillary break. Since this is the first time experimental waveforms for the embedded sensor geometry have been recorded, the sensor must be calibrated by comparing experimental sensitivity to the theoretical wave field. Comparison of theoretical particle velocity history and sensor output voltage at the initial P-wave peak yields a particle velocity sensitivity of 2.34 V output per mm/s. The calculation was checked against an independent solution based on a modified Cagniard technique (Ruitenbeek et al., 1991), which yields the same calibration factor.

The major difference between theory and experiment is the extra downward pulse output by our sensor following the initial peak. Forward modeling could only duplicate this artifact by including a thin, very slow, surface layer underneath the source. Continued improvements in sensor design should eliminate this artifact. The other minor wiggles are assumed due to “imperfections of reality,” such as finite extent of the source, imperfect material, etc., and do not interfere with interpretation of the recorded signal. The “double-pulse”

FIG. 8. Comparison of measured and calculated waveforms due to surface step force. The sensor is embedded 152 mm below the surface. Source located on the surface at (a) 25 mm, (b) 50 mm, (c) 75 mm, (d) 100 mm, (e) 126 mm, (f) 152 mm.
should not interfere with quantitative interpretation of source kinematics because it can easily be accounted for through deconvolution. Since the "double-pulse" appears after the initial rise starts falling, the principle parameters for moment tensor inversion such as polarity of initial $P$- and $S$-wave arrival, rise time, and peak amplitude are not distorted.

B. Discussion of theoretical and recorded off-epicentral waveforms

A comparison of measured and calculated particle velocity time histories for a variety of source–receiver geometries is shown in Fig. 8. The sensor was buried in a constant position 152 mm below the horizontal surface, monitoring motions parallel to this exposed surface. A step force was input by capillary breaks every 25 mm from epicenter, from 25 to 152 mm. These six positions correspond to azimuths of 9.5°, 18.5°, 26.5°, 33.7°, 40°, and 45°. As the azimuthal angle increases, more energy is partitioned from the $P$ wave to the $S$ wave, as is evidenced by the changing ratio of $P$- and $S$-wave peak amplitudes.

For all signals recorded, the rise times and the ratios of $P$-wave to $S$-wave amplitude are very similar for theory and experiment. The fit of the $S$ wave is exact for the 75-mm offsets and within 76% for the 100-, 126-, and 152-mm offsets. For the epicenter and the 25- and 50-mm offsets the Green’s function method provided a better fit than the viscous model (Kennett, 1983). The quality of these results leads to the conclusion that our sensor provides a quantitative time history of the particle motion inside the test block.

C. Evolution of the head wave

For the geometry used in this experiment, the head wave should appear when the angle between source and receiver was greater than or equal to the critical angle $\gamma_c$ (Pao and Gajewski, 1977; Cerveny and Ravindra, 1971)

$$\gamma_c = \sin^{-1}\left(\frac{v_s}{v_p}\right).$$

FIG. 9. Evolution of the head wave, experiment versus theory. The sensor is embedded 152 mm below the surface. Source position on surface (a) 107 mm; (b) 112 mm; (c) 117 mm; (d) 122 mm from epicenter.
As a test of the accuracy of modeling assumptions made in this paper, a step force was input at four offsets bracketing the expected initial arrival offset. These signals are shown in Fig. 9(a)–(d) compared with the actual particle velocity time histories recorded within the test block. For the host material used for this experiment \( \gamma_c = 37.3^{\circ} \), and the head wave should start to appear at a source offset of 116 mm. The headwave first appears at the 117-mm offset expected from theory. Note that for both the 117 [Fig. 9(c)] and 122 mm [Fig. 9(d)] offsets the duration and amplitude of the headwave is identical for theory and experiment.

VI. APPLICATION OF THE EMBEDDED SENSOR

We are working in conjunction with Shell Exploration and Production Research Company, to conduct laboratory-scale experiments of hydraulic fracture propagation (Dudley et al., 1995). To gain a better understanding of the fracture mechanisms occurring, the test samples are being monitored with an array of acoustic emission and active imaging sensors. Sandstone samples are loaded into a polyaxial load frame located at Shell E&P in Houston, TX, so that independent confining stresses can be applied to the sample in the principle directions, pore water pressure can be controlled, and fluid can be injected into the sample to induce fracture. In this application, the driving consideration for using embedded sensors is the inaccessibility of the specimen surface.

During hydrofracture testing at Shell E&P the embedded sensor was subjected to difficult environmental conditions. The test specimen was subjected to confining effective pressures of about 3.5 MPa, and could be saturated with brine with a pore pressure of 1 MPa. Embeddible sensors have been glued inside several specimens which were then loaded to failure and the sensors retrieved by overcoring. The only damage to the retrieved sensors was some tearing of the brass film wearplate.

The embedded sensor is an ideal laboratory tool for material science investigations. Potential applications for embedded sensors include: monitoring of concrete dams and highway bridges, monitoring of mines, tunnels, or other geologic structures, and monitoring and basic research on the performance of new polymer and composite structures.

Various improvements to the sensor containment are being investigated. The stiffness of the PZT ceramic leads to an impedance matching problem when it is used to monitor relatively low stiffness materials. We are currently investigating other soft elastomers such as urethanes which will not exhibit glass transition at frequencies of interest.

VII. CONCLUSIONS

Construction details of the embeddible sensor have been presented to give the reader insight into the device. The sensor is based on the NIST conical piezoelectric element, measures particle velocity at a point, and is sensitive only to motion parallel to the sensor long axis. The device is small, 38 mm long and 16 mm in diameter, which allows convenient mounting both inside or on a specimen.

This paper presents a unique view of motion inside a solid. For the first time, particle velocity time histories inside a body due to external forces have been accurately recorded. Comparison with theory shows that the sensor is an extremely faithful transducer of particle motion. When embedded the output is proportional to particle velocity and gives a sensitivity of 2.34 V per mm/s. Comparison with theory and NIST calibration prove the sensor to be an excellent transducer of surface displacement with a calibrated sensitivity of 2.8 V per nm.

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